Analytical Evaluation of Electrode Potential Drops in MHD Plasmas

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Introduction

RECENTLY, some investigations[1-4] into the possibility of obtaining the required current density in the diffusion mode of current conduction to alleviate the problem of electrode erosion in the arcing mode in MHD plasmas have been reported. These investigations show that the cathode drop is usually much higher than the anode drop and, for thermionically nonemitting electrodes, the cathode drop accounts for nearly 90% of the total applied potential between the two electrodes. On the other hand, in the arcing mode, the potential drops at the cathode and anode surfaces are considered to be caused mainly by the ohmic loss;[5] hence, the two drops are nearly equal.

In the diffusion mode, potential drops cannot be explained by ohmic loss only because they are different at the cathode and anode. An analysis has been made by Okazaki et al.[6] which is, however, applicable only to the anode region. Sodha et al.[2] attempted to explain the observed potential drops at cathode and anode by using the concept of electrostatic sheath. The analysis in Ref. 2 assumes that each of the two electrostatic sheaths contain only one type of charge carriers: ions in the cathode sheath and electrons in the anode sheath. This concept is able to explain the potential drops very well provided the cathode is thermionically nonemitting. However, when some low ionization potential material, such as the seed (e.g., K_2CO_3) in MHD generators, gets deposited on the surface of the cathode, there is a thermionic emission of electrons beyond a critical cathode temperature and current density increases. For emitting cathodes, the observed variation of anode potential drop with cathode temperature cannot be explained reasonably by the sheath theory, as can be seen from the results of Raju et al.[3] This may be attributed to the assumption that the anode sheath is devoid of positive ions which is not true because the ions have a much lower mobility than the electrons and, therefore, cannot completely move out of the sheath. In the present communication, a new model is proposed for the evaluation of potential drops at the electrodes in the diffusion mode.

Analysis and Results

The present model is based on the following:

1) There is an electrostatic sheath near the cathode that is devoid of electrons. In the diffusion mode, the electrons move out of the cathode sheath due to their high mobility. Therefore, one can assume the cathode sheath contains positive ions only, as the charge carries.

2) The electrostatic sheath drop near the anode is negligible. Depletion of ions from the region very close to the anode is negligible because of their low mobility. Therefore, the electrostatic sheath near the anode may be much less resistive than the cathode sheath. Therefore, one can neglect the sheath loss near the anode without much loss in the accuracy. As a consequence of this, the potential drop in the anode region is mainly due to ohmic loss.

From the above, it can be concluded that the drop in the potential at the cathode arises mainly in the electrostatic sheath, while at the anode, ohmic loss only can be considered. This differs from other theories which consider the same mechanism to be dominant at both the electrodes: it is either electrostatic sheath loss or ohmic loss.

The cathode region consists of two parts: the electrostatic sheath and ambipolar region. Most of the temperature difference between the electrode and main plasma is assumed to drop linearly in a very small region near the electrode, called the thermal sheath. Thickness of the thermal sheath is calculated by matching the heat flux entering the electrode from the plasma with the heat flux leaving the electrode. The current density inside the electrostatic sheath is equal to the sum of random ion current density at the sheath edge and the thermionic emission current density. The ion number density, required for calculating the random ion current density, is evaluated by matching the flux of charge carriers from the ambipolar region with the flux reaching the cathode. The potential drop in the cathode electrostatic sheath V_c^1 and ambipolar drop V_{amb} are given by the following expressions (Raju et al.[3])

$$V_c^1 = \int_0^{\delta_c} \left[\int_x^{\delta_c} (\sqrt{8} n_{is} K_B T_0 / \ell \epsilon) \mathrm{d}x^1 \right]^{1/2} \mathrm{d}x \tag{1}$$

$$V_{\text{amb}} = -(K_B T_0 / e) \cdot \ell n (n_{e\delta} / n_{es0}) \tag{2}$$

where δ_c is the thickness of the cathode electrostatic sheath, n_{is} the Saha's equilibrium ion density at the sheath edge, K_B the Boltzmann's constant, T_0 the plasma temperature, $\ell^2 = 2\mu_i K_B T_0 / (1 \cdot 1 \times 10^{-20} n_{es0}^2 T_0^{-4 \cdot 5}) n_{es0}$ the main plasma density, $n_{e\delta}$ the electron/ion density at the sheath edge after ambipolar diffusion, e the electronic charge, ϵ the permittivity of the free space, and μ_i the ion mobility. The total potential drop V_c in the cathode region is

$$V_c = V_c^1 + V_{\text{amb}}$$

Anode potential drop V_a can be written as

$$V_a = \int_0^{\delta_{T_a}} E_\chi \mathrm{d}_\chi \tag{3}$$

where δ_{T_a} is the anode temperature boundary layer thickness, and E_χ the electric field at a point χ in the boundary layer.

In the anode potential drop calculation, there may be Debye sheath of very small thickness which can be neglected compared to ohmic region. This leads to a very simplified approach without an appreciable loss in accuracy. Therefore, using ohm's law $E_\chi = J/\sigma$ one obtains

$$V_a = \int_0^{\delta_{T_a}} (J/\sigma) \mathrm{d}\chi \tag{4}$$

where σ is the electrical conductivity at a point χ, and J is the current density which is constant in the diffusion mode of current conduction. The equation for V_a requires electrical conductivity (σ) distribution inside the boundary layer, which, in turn, requires the electron density distribution. Usually, the electron density distribution inside the boundary layers is assumed to be given by Saha's equilibrium. A better estimation of charge density distribution should include the effects of diffusion and recombination which involves the solving of continuity and Poisson equation in conjunction with the momentum balance equation. This is a very complicated process and requires lengthy numerical calculations. Therefore, to illustrate the effectiveness of calculating anode potential drop through ohmic loss only, a very simple analytical expression of anode potential drop shall be derived. The electrical conductivity distribution inside the boundary layer is expressed as a power law in temperature[7]

$$\sigma/\sigma_0 = (T/T_0)^\alpha \tag{5}$$

where σ_0 and T_0 are, respectively, the electrical conductivity and temperature at the channel axis.

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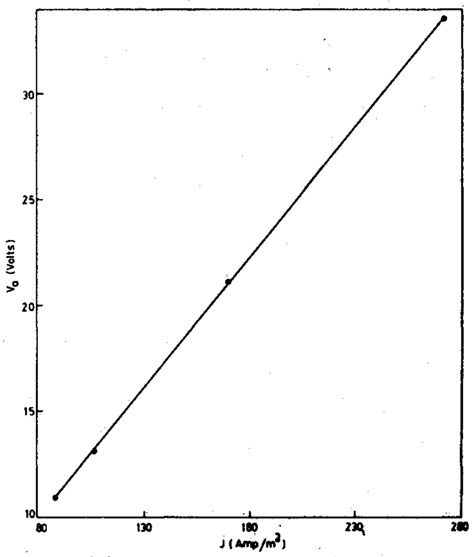

Fig. 1 Experimental variation of anode potential drop with a current density for an applied potential difference of 100 V.

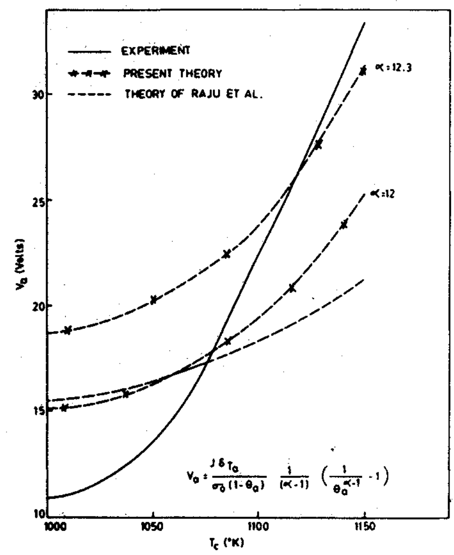

Fig. 2 Variation of anode potential drop with a cathode temperature for an applied potential difference of 100 V.

Since most of the temperature difference inside the thermal boundary layer is assumed to drop in a very small region of thickness δ_{T_a} the temperature T is given by

$$T = T_a + Y/\delta_{T_a}(T_0 - T_a) \tag{6}$$

where T_a is the temperature of the anode surface in contact with the plasma.

Using Eqs. (5) and (6) in Eq. (4), one obtains

$$V_a = \frac{J\delta_{T_a}}{\sigma_0(1-\theta_a)} \frac{1}{(\alpha-1)} \left(\frac{1}{\theta_a^{\alpha-1}} - 1\right) \tag{7}$$

where

$$T/T_0 = \theta \text{ and } T_a/T_0 = \theta_a$$

The total potential difference between the electrodes is the sum of cathode potential drop V_c and anode potential drop V_a

$$V = V_c + V_a \tag{8}$$

Calculations for potential drops have been made for seeded (with K_2CO_3) combustion products of liquefied petroleum gas and oxygen. Composition of the combustion products for the system has been evaluated and is used in calculating various parameters such as electron density n_{es0} mobility of charged particles and main plasma electrical conductivity σ_0. The experimental curves have been drawn by using data of Raju et al.[3] Limited calculations are presented here to illustrate the validity of the present model and to show the qualitative improvement in the potential drop calculations. Figure 1 shows the experimental observation of the variation of the current density with anode potential drop V_a. Interestingly, it is seen that the current density varies linearly with anode potential drop which is a similar behavior to that predicted by Eq. (7). Therefore, it supports the assumption that one can neglect the electrostatic sheath losses and consider ohmic losses only for calculation of the anode potential drop. Swifthook and Wright[7] suggested that α in Eq. (5) varies between 13 and 10 for the electrical conductivity values in the range of 0.1 mhos/m to 100 mhos/m. Calculation of the exact value of α or the distribution of α inside the boundary layer requires the electron density distribution inside the boundary layer. This can be obtained by solving the coupled continuity, energy, and momentum balance equations. However, in the present work, rather simplified calculations have been made for differnt values of α taking it as constant throughout the boundary layer, since the purpose of the communication is only to show the effectiveness of the present approach. A comparison of the variations of the theoretical curves with the experimental curve in Fig. 2 reveals that the present calculations show much better behavior than the earlier calculation of Raju et al.[3] It is seen that at high cathode temperatures, the present theory for $\alpha = 12.3$ is in agreement, even quantitatively, with the experimental observation.

For a more exact calculation, one can take into account the exact dependence of electron density on temperature and nonlinear temperature distribution inside the electrode boundary layer. Moreover, recombination and space charge effects near the cathode may become important for thermionically emitting electrodes.

Conclusion

From the present work it is concluded that in diffusion mode of current conduction, the potential drops near the electrodes can be calculated to a reasonable level of accuracy by considering electrostatic sheath at the cathode and only ohmic loss at the anode.

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